# Information flow and information production in a population system

S. C. Nicolis*

*Mathematics Department, Uppsala University, P.O. Box 480, SE-751 06 Uppsala, Sweden*

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An approach aiming to quantify the dynamics of information within a population is developed based on the mapping of the processes underlying the system's evolution into a birth and death type stochastic process and the derivation of a balance equation for the information entropy. Information entropy flux and information entropy production are identified and their time-dependent properties, as well as their dependence on the parameters present in the problem, are analyzed. States of minimum information entropy production are shown to exist for appropriate parameter values. Furthermore, uncertainty and information production are transiently intensified when the population traverses the inflexion point stage of the logisticlike growth process.

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## I. INTRODUCTION

A population of group-living organisms is often led to make collective decisions concerning internal organization and communication with the external environment [1,2]. During this process, we witness information transduction in the sense that group members start communicating with each other until eventually a behavior encompassing the population as a whole is established.

The objective of this paper is to propose an approach to information dynamics within a population based on entropy-like quantities and their correlates. To this end, we adopt a setting in which the principal variable of interest is the number of individuals in the population, discarding at this stage "internal" states corresponding to, e.g., options or signaling conventions to be adopted. In such a setting, information and its correlates are associated to different ways that the population may be reshuffled according to the values of the intrinsic parameters and the environmental constraints. To arrive at a quantitative view of this process, we map the evolution of the population into a birth and death type Markov process [3,4] and derive a balance equation for the information (Shannon) entropy associated to the process [5,6]. This balance equation then allows one to identify an information entropy flux and an information entropy production, the latter playing a role analogous to the thermodynamic dissipation associated to physicochemical processes out of the state of equilibrium [7,8]. We analyze the dependence of these quantities on time, population size, and other parameters present in the problem and bring out a number of properties that provide useful characterizations of the system at hand.

The population model is introduced in Sec. II, where a macroscopic (mean-field) analysis serving as a reference for the subsequent developments is also carried out. Section III is devoted to the probabilistic (Markov) formulation and to the analytic evaluation of the asymptotic (steady-state) solution. The derivation of the information entropy balance is reported in Sec. IV. The time-dependent properties and the parameter dependencies of the principal quantities of interest are analyzed in Sec. V, and the main conclusions are summarized in Sec. VI.

*snicolis@math.uu.se

## II. MODEL AND MACROSCOPIC ANALYSIS

Throughout this paper, we will be concerned with the classical problem of logistic growth where, in addition, an exchange of population between the subsystem of interest and the external medium is allowed [9]. The rates $v$ of the various kinetic steps associated to these processes will be modeled as follows:

(i) Population growth: $v_1 = k_1 a x$, where $x$ is the population density within the system of interest, $a$ is the density of available resources, and $k_1$ is a kinetic factor.

(ii) Saturation: $v_1' = k_1' x^2$, where $k_1'$ is a kinetic factor.

(iii) Outflow of individuals from the system of interest to the external medium: $v_2 = k_2 x$, where $k_2$ is a kinetic factor.

(iv) Inflow of individuals from external medium: $v_2' = k_2' b$, where $b$ is the population density within the external medium and $k_2'$ is a kinetic factor.

The balance equation for the population density $x$ takes thus the form

$$\frac{dx}{dt} = v_1 - v_1' - v_2 + v_2' = k_1 a x - k_1' x^2 - k_2 x + k_2' b. \tag{1}$$

In population dynamics, it is customary to cast such equations in a form displaying the growth rate $r$ and the carrying capacity $K$ of the ecosysttem,

$$\frac{dx}{dt} = r x \left(1 - \frac{x}{K}\right) - k_2 x + k_2' b. \tag{2a}$$

Comparing with the original form leads to the following expression of $r$ and $K$:

$$r = k_1 a, \quad K = \frac{k_1 a}{k_1'}. \tag{2b}$$

The steady-state solutions $x_s$ of Eq. (1) are given by

$$k_1' x_s^2 + (k_2 - k_1 a) x_s - k_2' b = 0. \tag{3}$$

There is, thus, only one physically acceptable solution as long as $k_2' b \neq 0$. Figures 1(a) and 1(b) depict the time evolution of $x$ as given by Eq. (1) for the initial condition $x(0) = 0.01$ and two different values of $k_2$ and $k_2'$. The solutions approach the asymptotic value $x_s$ on a characteristic time scale determined by the derivative of the right-hand side of Eq. (1) with respect to $x$:

$$\tau_x = (-2k_1' x_s + k_1 a - k_2)^{-1}. \tag{4}$$

 

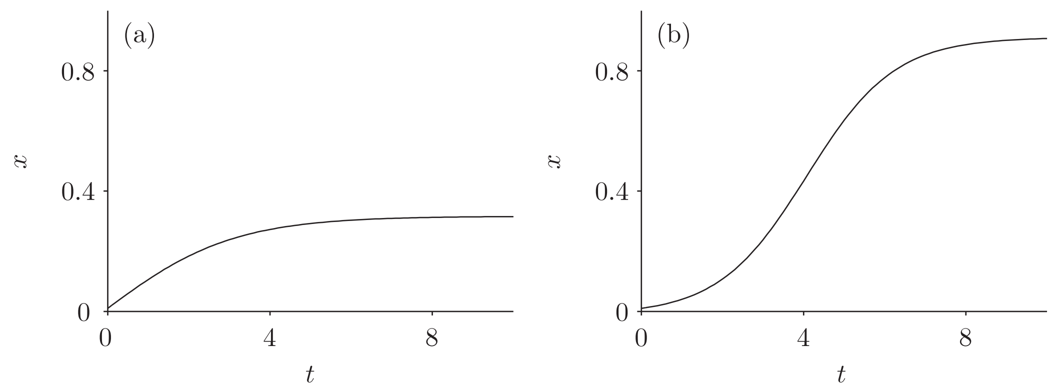


FIG. 1. Time dependence of $x$ [Eq. (1)] with $a = 1$, $b = 0.1$, $k_1 = k_1' = 1$, and $k_2 = k_2' = 1$ (a), $k_2 = k_2' = 0.1$ (b). Initial condition is $x(0) = 0.01$.

We notice that, as the rates of outflux and influx become smaller and as long as the initial population is much less than the asymptotic one, the evolution tends to follow a sigmoidal curve familiar from the solution of the logistic (Verhulst) equation in a closed system [9].

## III. MAPPING INTO A MARKOV PROCESS: THE BIRTH AND DEATH MASTER EQUATION

In this section, we adopt a more mechanistic point of view in which the four different terms contributing to Eq. (1) are viewed as the result of transitions between states characterized by different numbers $X$ of individuals within the subsystem of interest. The rates of these transitions depend on the instantaneous values of $X$ and, as a result, fluctuations around the macroscopic state $x = \overline{X}/N$, where $\overline{X}$ is the average value of $X$ and $N$ is an extensivity factor representing the area available or the total allowable number of individuals, are automatically incorporated in the description. We will adopt a Markovian approximation, i.e., that the system's memory does not extend beyond transitions between a state $X$ at time $t$ and a state $X \pm 1$ at time $t + \Delta t$, with $\Delta t$ being small compared to the characteristic time scales present in the problem.

We now write out the four transition probabilities per unit time and area:

(i) First term in Eq. (1):

$$w_1(X-1 \to X) = k_1 a \frac{X-1}{N}. \tag{5a}$$

(ii) Second term in Eq. (1):

$$w_1'(X \to X-1) = k_1' \frac{X(X-1)}{N^2}. \tag{5b}$$

(iii) Third term in Eq. (1):

$$w_2(X+1 \to X) = k_2 \frac{X+1}{N}. \tag{5c}$$

(iv) Fourth term in Eq. (1):

$$w_2'(X \to X+1) = k_2' b. \tag{5d}$$

In writing these relations, we assumed that the resources (term in $a$) and the reservoir (term in $b$) are not fluctuating. Furthermore, processes corresponding to the first two and the last two terms in Eq. (1) are modeled as pairs of direct and reverse transitions, respectively. Finally, the quadratic term in (1) associated with the $x$ dependence of the growth rate has been modeled as a second order rate process involving encounters between two individuals, the frequency of which is proportional to the number of pairs that can be formed in a population of size $X$.

Let $P(X,t)$ be the probability to find $X$ individuals within the subsystem at time $t$. By utilizing the Markov character of the process, one may write the evolution equation of $P$ in the form of a master equation [3,4]

$$\begin{aligned}\frac{dP(X,t)}{dt} = {} & N\{w_1(X-1 \to X) + w_2'(X-1 \to X)\} \\ & \times P(X-1,t) + N\{w_1'(X+1 \to X) \\ & + w_2(X+1 \to X)\}P(X+1,t) \\ & - N\{w_1(X \to X+1) + w_2(X \to X-1) \\ & + w_1'(X \to X-1) + w_2'(X \to X+1)\}P(X,t), \\ & 1 \leqslant X \leqslant N-1\end{aligned} \tag{6a}$$

and

$$\frac{dP(0,t)}{dt} = k_2 P_1 - k_2' b N P_0, \tag{6b}$$

$$\begin{aligned}\frac{dP(N,t)}{dt} = {} & \{k_1 a(N-1) + k_2' b N\}P(N-1,t) \\ & - \{k_1'(N-1) + k_2 N\}P(N,t),\end{aligned} \tag{6c}$$

where the factor $N$ in front of the $w$'s accounts for the property of extensivity.

Equations (6) admit a unique steady-state solution $P_s$, which, under the boundary conditions of zero probability flux at $X = 0$ and $N$, satisfies the recurrence relation

$$\begin{aligned}&\{w_1(X-1 \to X) + w_2'(X-1 \to X)\}P_s(X-1) \\ &\quad = \{w_1'(X \to X-1) + w_2(X \to X-1)\}P(X)\end{aligned} \tag{7}$$

leading to the explicit expression

$$P_s(X) = P_s(0) \prod_{j=1}^{X} \frac{w_1(j-1 \to j) + w_2'(j-1 \to j)}{w_1'(j \to j-1) + w_2(j \to j-1)}, \tag{8}$$

where $P_s(0)$ is evaluated from the normalization condition $\sum_X P_s(X) = 1$.

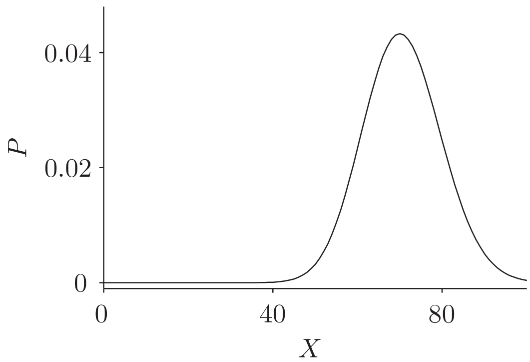


FIG. 2. Steady-state probability distribution as obtained from the numerical solution of the master equation (6). Parameter values are $k_1 = k_1' = k_2 = k_2' = 1$, $b = 0.5$, $a = 1$, and $N = 100$.

Figure 2 depicts $P_s$ a a function of $X$ as obtained from Eq. (8) for a maximum population size $N = 100$, in excellent agreement with the result obtained from the numerical solution of the initial value problem of the full master equation (6) using an implicit integration scheme (not shown). The distribution is nearly Gaussian in the vicinity of its maximum and subsequently falls to zero on both sides in an asymmetrical fashion, owing to the finite size of the population and to the presence of a boundary at $X = 0$.

## IV. ENTROPY ANALYSIS: INFORMATION FLOW AND INFORMATION PRODUCTION

As stressed in the preceding section, the probabilistic description accounts automatically for the variability that may be present in the process of interest and, hence, for the uncertainty of the observer to deduce the state of the system on the basis of the knowledge of the probability distribution $P(X,t)$. A natural measure of variability and uncertainty, on the grounds of its convexity and additivity properties, used widely in information theory and in physical sciences is the information entropy [5,6]

$$S(t) = -\sum_X P(X,t) \ln P(X,t). \tag{9}$$

To see how information entropy is behaving during different stages of the evolution, we differentiate both sides of Eq. (8) with respect to time and replace $dP/dt$ in the right-hand side of the master equation (6). We obtain

$$\begin{aligned}\frac{dS}{dt} = \frac{1}{2}\Bigg\{&\sum_{X=1}^{N}[w_1(X-1\to X)P(X-1)\\ &- w_1'(X\to X-1)P(X)]\ln\frac{P(X-1)}{P(X)}\\ &+\sum_{X=0}^{N-1}[w_2(X+1\to X)P(X+1)\\ &- w_2'(X\to X+1)P(X)]\ln\frac{P(X+1)}{P(X)}\Bigg\}.\end{aligned} \tag{10}$$

By adding and subtracting in the two terms of the right-hand side the logarithm of the ratio of $w_1$ and $w_1'$ and of $w_2$ and $w_2'$, respectively, one is led to decompose $dS/dt$ into two types of terms [7,8]

$$\frac{dS}{dt} = J + \sigma, \tag{11}$$

where $\sigma$ is positive definite,

$$\begin{aligned}\sigma = \frac{1}{2}\Bigg\{&\sum_{X=1}^{N}[w_1(X-1\to X)P(X-1)\\ &- w_1'(X\to X-1)P(X)]\ln\frac{P(X-1)w_1(X-1\to X)}{P(X)w_1'(X\to X-1)}\\ &+\sum_{X=0}^{N-1}[w_2(X+1\to X)P(X+1)\\ &- w_2'(X\to X+1)P(X)]\ln\frac{P(X+1)w_2(X+1\to X)}{P(X)w_2'(X\to X+1)}\Bigg\},\\ \geqslant 0&\end{aligned} \tag{12}$$

and $J$ is given by

$$\begin{aligned}J = \frac{1}{2}\Bigg\{&\sum_{X=1}^{N}[w_1(X-1\to X)P(X-1)\\ &- w_1'(X\to X-1)P(X)]\ln\frac{w_1'(X\to X-1)}{w_1(X-1\to X)}\\ &+\sum_{X=0}^{N-1}[w_2(X+1\to X)P(X+1)\\ &- w_2'(X\to X+1)P(X)]\ln\frac{w_2'(X\to X+1)}{w_2(X+1\to X)}\Bigg\}.\end{aligned} \tag{13}$$

By analogy with thermodynamics [10], we refer to $\sigma$ and $J$ as the information production and the information flux, respectively. We obtain then a picture of the system's evolution as a process in which information is continuously produced by the dynamical processes within the system (term $\sigma$) while being at the same time exchanged with the outside world (term $J$). Within the minimal setting adopted as described in the Introduction, this provides us with a way to quantify the concept of information transduction by a group of interacting individuals and, at the same time, with an alternative way to characterize the complexity of its dynamics.

It should be pointed out that, just as in irreversible thermodynamics, the decomposition of the rate of change of $S$ into a production and a flux part is not unique. The logic followed in the decomposition adopted here is that, first, it is the most natural one ensuring that the production part is positive definite. Furthermore, if it turned out that the system's evolution could be cast into elementary steps expressible in terms of thermodynamic rates and forces, then, in the limit of large population size $N$, expression (12) should reduce to the thermodynamic entropy production $\sigma_{\mathrm{th}}$. In the present problem, this correspondence is indeed legitimate, since the steps leading to Eq. (1) are isomorphic to the rate processes

$$\begin{aligned}A + X &\underset{k_1'}{\overset{k_1}{\rightleftharpoons}} 2X,\\ X &\underset{k_2'}{\overset{k_2}{\rightleftharpoons}} B,\end{aligned} \tag{14}$$

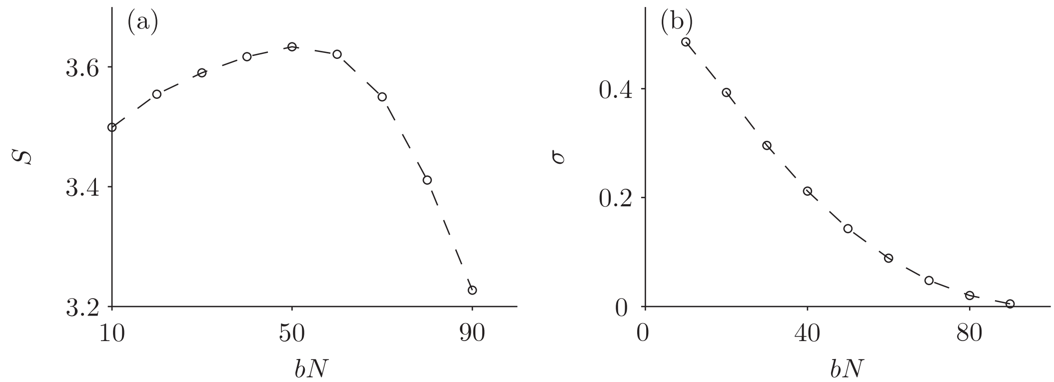


FIG. 3. Dependence of $S$ [Eq. (9)] and $\sigma$ [Eq. (12)] on parameter $bN$ in the steady state. Other parameters are kept at the same values as in Fig. 2.

leading to the the following expression for the thermodynamic entropy production [10] :

$$\sigma_{\rm th} = (k_1 a x - k_1' x^2) \ln \frac{k_1 a}{k_1' x} + (k_2 x - k_2' b) \ln \frac{k_2 x}{k_2' b}. \quad (15)$$

As seen in the next section, the value of $\sigma$ deduced from expression (12) tends indeed to $\sigma_{\rm th}$ if $N$ is sufficiently large, thereby further justifying *a posteriori* our choice of the production term $\sigma$. This, in turn, justifies qualifying the second part in the decomposition of Eq. (11) as flux. An additional reason is that $J$ is a weighted sum of probability fluxes [terms in square brackets in Eq. (13), see also comments below]. Notice that since we are dealing with a spatially well-mixed system, the familiar expression of flow as a divergence of a current does not find here an analog.

Going back to relations (11)–(13), in the stationary state, $dS/dt = 0$ and $J$ and $\sigma$ cancel each other, which amounts to saying that information production is sustained by a negative information flux. We notice that $J$ and $\sigma$ are nonvanishing as long as the terms in square brackets in Eqs. (12) and (13) are nonzero. Now these terms represent the probability fluxes associated to the transitions $X-1 \leftrightarrows X$ and $X+1 \leftrightarrows X$. They are, thus, nonzero as long as the direct $(X \pm 1 \to X)$ and the reverse $(X \to X \pm 1)$ process do not cancel each other in the stationary state, a property that plays a role similar to the deviation from the state of thermodynamic equilibrium in a physicochemical context. If the process associated to $w_1$ and $w_1'$ were the only one present (and likewise for the process associated to $w_2$ and $w_2'$), the stationarity condition [Eq. (7)] would entail the vanishing of the probability fluxes and hence of $\sigma$ and $J$ as well. In other words, the existence of at least two coupled processes is a necessary (although not sufficient) condition for information to be permanently produced within the system.

## V. TIME AND PARAMETER DEPENDENCE OF INFORMATION ENTROPY AND INFORMATION PRODUCTION

Figures 3(a) and 3(b) summarize the dependence of information entropy $S$ and information production $\sigma$ in the stationary state as a function of parameter $b$, keeping the other parameters to the values corresponding to Fig. 2. As can be seen, $\sigma$ decreases, tending to zero as $b$ tends to the value $b_{\rm eq} = k_1 k_2 a / k_1' k_2'$. Now, in the kinetic scheme (14), the state of thermodynamic equilibrium is achieved if, in each of the two elementary steps, the rate of the forward transition is counteracted by the rate of the backward one. Fixing $a$ and the rate constants once for all one obtains

$$k_1 a x_{\rm eq} = k_1' x_{\rm eq}^2, \quad k_2 x_{\rm eq} = k_2' b_{\rm eq} \quad (16)$$

or, eliminating $x$ between the two relations,

$$k_1 k_2 a = k_1' k_2' b_{\rm eq}. \quad (17)$$

This yields a value of $b_{\rm eq}$, which is exactly equal to the aforementioned value for which $\sigma$ vanishes. One can check straightforwardly that, under the same conditions, the thermodynamic entropy production $\sigma_{\rm th}$ vanishes as well. A different way to express this result is as follows: By summing the two steps in (14), one obtains an overall transition from $A$ to $B$, whose forward rate $k_1 k_2 a$ and backward rate $k_1' k_2' b$ become equal in the state of equilibrium. In short, information is produced within the system only to the extent that the ongoing processes do not balance each other individually. Interestingly, the entropy $S$ presents a maximum for an intermediate value of $b$ far from the equilibrium one. On inspecting the structure of $P_s(X)$, one sees that the distribution is becoming broader in the vicinity of this value, entailing that uncertainty is thereby enhanced.

The behavior of $S$ and $\sigma$ with respect to parameter $k_1$ for fixed values of the other parameters is represented in Figs. 4(a) and 4(b). This parameter provides a measure of the relative importance of processes 1 and 2 and is seen to lead to a minimum of $\sigma$ close to zero at a value $k_{1m}$, where process 1 equilibrates, in the mean, with its inverse. Varying $k_1$ on both sides of this value leads to increasing values of $\sigma$. Furthermore, $S$ goes through an inflexion point in the vicinity of this value and subsequently presents a mild overshoot. We notice that both $\sigma$ and $S$ behave asymmetrically around $k_{1m}$. Decreasing $k_1$ [and thus the carrying capacity $K$, see Eq. (2b)] leads to significantly larger values of information production than when $k_1$ is increased. On the other hand, information entropy decreases significantly with decreasing $k_1$ but increases less rapidly when $k_1$ is increased, tending to a plateau in this range of values. Stated differently, as the resources become limited, the group tends to become more coherent and to

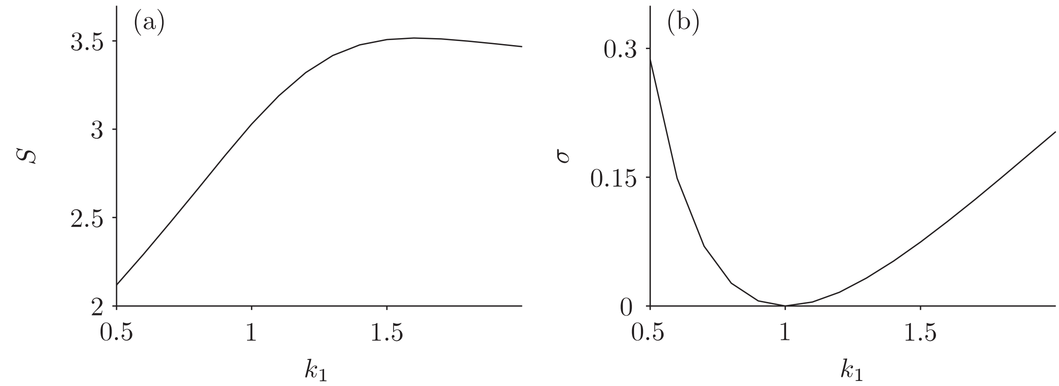


FIG. 4. Dependence of (a) $S$ and (b) $\sigma$ on parameter $k_1$ for $N = 100$. Other parameters are $k_1' = k_2 = k_2' = 1$ and $a = b = 1$.

produce information more intensely, an unexpected and in some respects counterintuitive trend. Finally, as expected, both $S$ and $\sigma$ are monotonically increasing functions of size $N$, keeping all other parameters fixed (not shown).

Let us turn next to time-dependent properties. Figures 5(a) and 5(b) depict the time dependence of $S$ and $\sigma$ for an initial condition corresponding to a uniform occupation of all states $P(X,0) = 1/(N+1)$. Both $S$ and $\sigma$ are seen to decrease monotonically in time. The system thus attains, time going on, a state of both minimum entropy and minimum information production compatible with the constraints present. In a sense, starting with a situation of maximum uncertainty, the

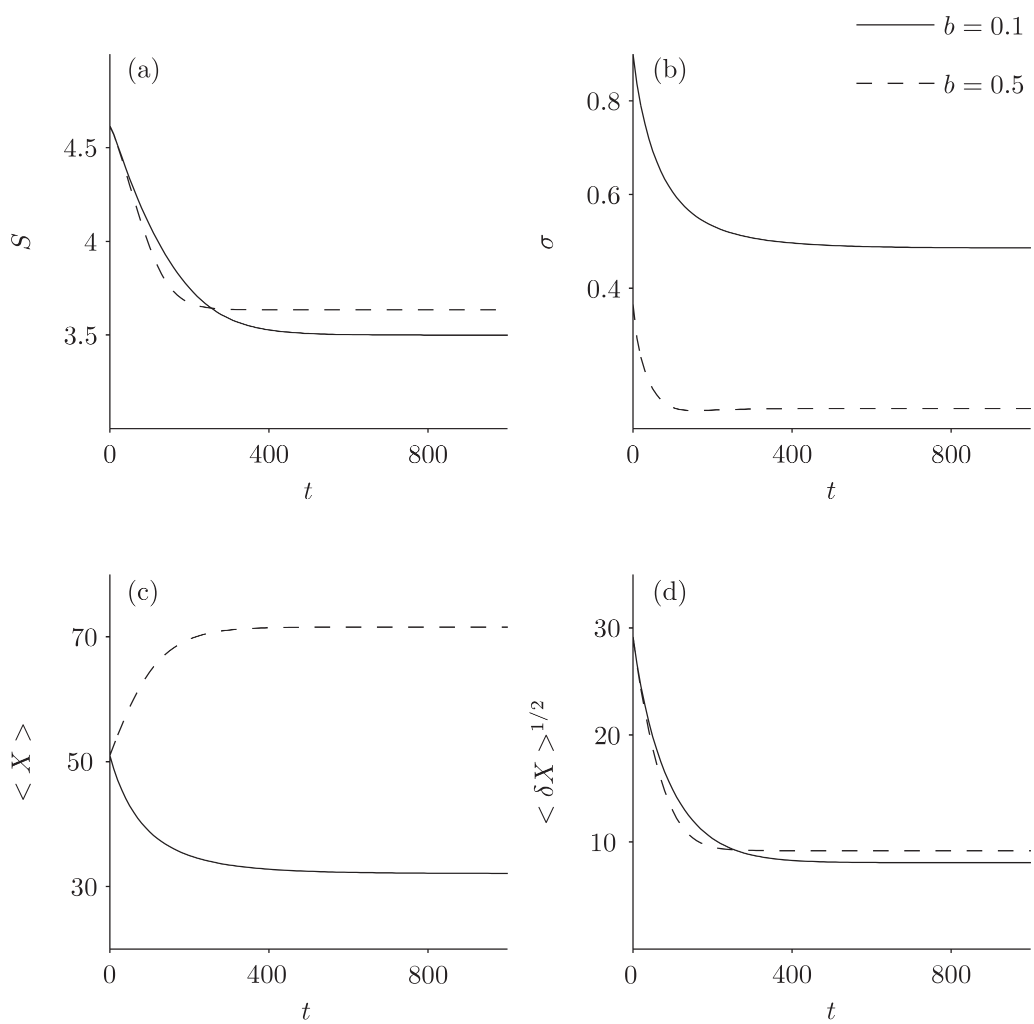


FIG. 5. Time evolution of (a) $S$, (b) $\sigma$, (c) $\langle X \rangle$, and (d) $\langle \delta X^2 \rangle^{1/2}$ for two different values of $b$ and for an initial probability of uniform occupation of all states. Other parameter values are $k_1 = k_1' = k_2 = k_2' = 1$, $a = 1$, and $N = 100$.

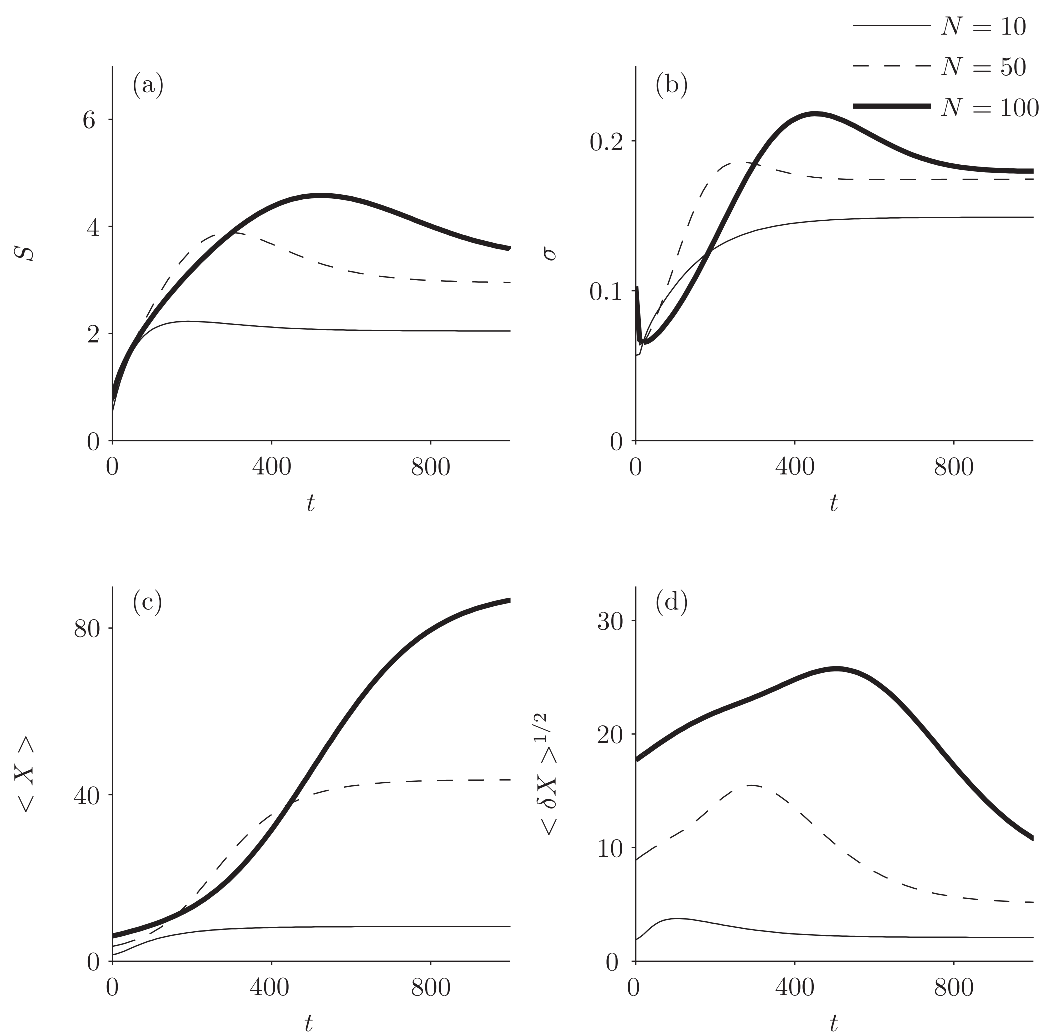


FIG. 6. Time dependence of (a) $S$, (b) $\sigma$, (c) $\langle X\rangle$, and (d) $\langle \delta X^2\rangle^{1/2}$ for different values of the size $N$ and for an initial probability centered on state $X = 0$. Other parameters are $k_1 = k_1' = 1$, $k_2 = k_2' = 0.1$, $a = 1$, and $b = 0.1$.

dynamics leads to an increasingly sharp selection of the state of the system, which becomes practically identical [up to small fluctuations of $O(1/N)$] to the state predicted by the deterministic evolution laws. In parallel, in the course of this selection, the system gradually filters much of the ongoing information production until a minimal amount necessary for the sustainability of the final state is reached. The time dependencies of the expectation value $\langle X\rangle$ of the population and of the standard deviation $\langle \delta X^2\rangle^{1/2}$ around it as deduced from the master equation are shown in Figs. 5(c) and 5(d). They suggest that the behavior shown in Figs. 5(a) and 5(b) is to be correlated to the rapid tendency of $\langle X\rangle$ toward its final saturation value and to a concomitant rapid decrease of the fluctuations.

The situation is very different for initial conditions corresponding to a probability mass centered predominantly in state $X = 0$, emulating a classical population growth problem where, starting with few individuals, the group eventually reaches a finite saturation level. Specifically, we set $P(X,0) = \epsilon/N$ for $X$ from 1 to $N$ and $P(0,0) = 1 - \epsilon$ with $\epsilon \ll 1$. As seen in Figs. 6(a) and 6(b), for $\epsilon = 0.1$ and depending also on the size $N$, both $S$ and $\sigma$ may present nonmonotonic behaviors in the form of overshoots and undershoots before settling to their asymptotic values. This behavior turns out to be robust, subsisting for $\epsilon$ values up to 0.01. Contrary to Fig. 5, the dynamics leads here to the system from a regime of practically no uncertainty and practically no information production to regimes where information is gradually being produced until the system reaches its final state. In the course of this evolution, states of maximum uncertainty and maximum information production are transiently showing up, a trend that becomes quite pronounced for groups of sufficiently large size $N$ [dashed and bold lines in Figs. 6(a) and 6(b)]. The time dependencies of the expectation value $\langle X\rangle$ and of the standard deviation $\langle \delta X^2\rangle^{1/2}$ under these conditions, shown in Figs. 6(c) and 6(d), are likewise very different from those in Figs. 5(c) and 5(d). Specifically, the growth curve of $\langle X\rangle$ becomes increasingly sharp with increasing $N$, displaying a clear-cut inflexion point, while $\langle \delta X^2\rangle^{1/2}$ presents a maximum. Comparison with Figs. 6(a) and 6(b) shows that the maxima of $S$, $\sigma$ and $\langle \delta X^2\rangle^{1/2}$ occur at times close to the inflexion point while being slightly shifted to each other, the sequence being that the maximum of $\sigma$ is followed by the maximum of $\langle \delta X^2 \rangle^{1/2}$, which is in turn followed by the maximum of $S$. This suggests that the maxima of $S$ and $\sigma$ are to be correlated to the transient increase of variability around the mean; but, on the other side, being nonlinear functionals of the probability distribution, $S$ and $\sigma$ contain contributions of moments of an arbitrary order. This explains the shift between the times of occurrence of the maxima. In short, there is a definite correspondence between the rate of growth of the population, which attains its maximum at the inflexion point, and a transition in the way information is processed within the system.

## VI. CONCLUSIONS

Information theoretic concepts [5] in conjunction with the tools of the theory of stochastic processes [3] have been used extensively in mathematical and physical sciences in connection with, in particular, the foundations of irreversible thermodynamics [7,8,10]. In this paper, we developed an approach along similar lines, aiming to quantify information dynamics within a population. We considered the simplest nontrivial setting of logistic growth in an open system, mapped the dynamics into a birth and death Markov process, and derived a balance equation for the information (Shannon) entropy associated to the instantaneous probability distribution $P(X,t)$ of population size $X$. This allowed us to identify an information flux and an information production, to relate these quantities to the parameters present in the problem, and to assess their behavior during different stages of the evolution toward the asymptotic (steady) state. We have shown that information production, by construction a non-negative quantity, takes nonzero values as long as the rates of the different elementary steps in the process do not cancel each other, thereby maintaining the system in a state analogous to the nonequilibrium states familiar in thermodynamics. Depending on the initial preparation, the system could tend monotonously to a state of minimum entropy and information production or, on the contrary, present nontrivial behavior where entropy and information production go transiently through a maximum. The latter occurs at a time stage of the logistic growth process where the population size goes through an inflexion point, i.e., when its growth rate reaches a maximum. Furthermore, in the long time limit, information entropy and entropy production can display intricate behaviors with respect to the parameters. In particular, there exist states of minimum information production for special parameter values around which information is processed by the group in markedly asymmetric ways, the tendency being to enhance information production as the resources become limited. Such properties provide useful ways to characterize the system at hand, a conclusion similar to that reached in a recent paper by Andrae *et al.* [11] where the properties of information entropy production were studied in a three-competing species population model in the presence of mutations.

An interesting corollary of our analysis is that, as a rule, the steady states realized by the system are not states of extremal (be it minimum or maximum) information entropy production. This is due to both the presence of nonlinear steps in the dynamics and to the presence of nonequilibrium constraints. In particular, the validity of a "universal principle" of maximum dissipation in the most stable state that can be realized by a system, which is currently attracting attention in the literature [12,13], appears to be questionable in light of our results.

By its generic character, the analysis reported in this paper can be extended in different directions. A most natural case would be to consider more complex schemes accounting for the presence of several options as they occur, for instance, in a wide range of problems from social sciences and psychology to foraging in social insect populations [1,2,14,15]. Here, information exchange is expected to be subtler and considerably more involved than in the minimal setting adopted in this paper. The new issue arising is decision making. The information theory approach outlined in this paper would help to place this question in a dynamical perspective and to quantify the concepts of quality and rationality of this process, which to a large extent remain elusive [14,15]. Of interest would also be to assess the role of innovations [16] and of mutations as they occur in biological evolution [17]. Here, the appearance of new entities within the system is expected to open new communication and information exchange channels that could, under certain conditions, take over the existing ones and drive the system toward new modes of organization.

## ACKNOWLEDGMENTS

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